AN INERTIA CRITERION FOR COUPLED HEAT AND MASS TRANSFER PROCESSES

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New criterial relations are obtained with allowance for the influence of thermal gradient mass transfer, phase transitions and Kossovich number on the inertia of heat and mass transfer processes.

The generalized system of differential transport equations for the one-dimensional problem has the form [1]

$$\frac{\partial \Theta_i^{(m)}}{\partial \mathrm{Fo}_i} = \sum_{j=1}^{n} K_{ij}^{(m)} \left(\frac{\partial^2 \Theta^{(m)}}{\partial \xi^2} + \frac{\Gamma}{\xi} \frac{\partial \Theta^{(m)}}{\partial \xi} \right)_{i,\, j=1,\, 2, \ldots,\, n}. \tag{1}$$

Thus, for heat and mass transfer in capillary-porous materials

$$m,\ i,\ j = 1, 2;\ \Theta_1^{(1)} = T;\ \Theta_2^{(1)} = \Theta;$$
$$K_{11}^{(1)} = 1 + \mathrm{Ko^*Pn\,Lu};\ K_{12}^{(1)} = -\mathrm{Ko^*Lu};\ K_{21}^{(1)} = -\mathrm{LuPn};\ K_{22}^{(1)} = \mathrm{Lu}. \tag{2}$$

For a binary gas mixture

$$\Theta_1^{(2)} = t^*;\ \Theta_2^{(2)} = \rho^*;$$
$$K_{11}^{(2)} = 1 + \mathrm{DuSoLe};\ K_{12}^{(2)} = \mathrm{DuLe};\ K_{21}^{(2)} = \mathrm{LeSo};\ K_{22}^{(2)} = \mathrm{Le}. \tag{3}$$

The physical meaning of the parameters entering into K_{ij} was adequately explained in [1]. At the same time, certain questions concerning the criteria determining the rate of variation of the transfer potential fields require closer study. It was pointed out in [1] that the inertia criteria of the transfer potential fields may be represented by the Lewis and Lykov numbers Le and Lu. This is no doubt true in connection with unrelated transfer processes. For coupled transfer however, certain other criteria may be expected to influence the inertia of the corresponding fields. Thus, in heat and mass transfer in porous materials, the parameter ε and the Ko and Pn numbers must have an appreciable influence on the rate of establishment of steady moisture content and temperature profiles. Indirect evidence of this, noted in [1], is the lack of similarity of the temperature and potential fields when Lu = 1. Of course, a similar effect is also observed in connection with heat and mass transfer in binary mixtures.

We note that, even from a simple analysis of the initial differential equations (1), some positive information on the problem in question may be obtained. As a preliminary example, let us examine the system of equations for boundary-layer momentum and heat transfer in a zero-gradient flow of a viscous incompressible fluid over an isothermal plane surface (neglecting viscous dissipation and variation of the thermophysical properties of the liquid with temperature):

$$\frac{du}{dt} = \nu \frac{\partial^2 u}{\partial y^2}, \tag{4}$$

$$\frac{dT}{dt} = \frac{\nu}{\mathrm{Pr}} \frac{\partial^2 T}{\partial y^2}, \tag{5}$$

where d/dt is a substantive derivative.

It follows at once from an analysis of (4) and (5) that when Pr = 1 the temperature and velocity fields in the boundary layer are similar, the rates of establishment of steady-state u and T (for example, for an isothermal semi-infinite plate brought impulsively from rest into uniform motion) being the same [2]. We may Further determine, from a more detailed analysis, that when Pr > 1 the rate of establishment of a steady-state velocity field exceeds that for temperature, while the contrary holds true when Pr < 1.

On the basis of a similar kind of qualitative reasoning, which may be confirmed quantitatively by applying the integral methods of boundary layer theory to the solution of heat and mass transfer problems, it may be shown that, for given boundary conditions of the first kind, the criterion determining the inertia of the temperature and moisture content fields is a complex one of the type

$$\mu_1^{(m)} = \frac{K_{21}^{(m)} + K_{22}^{(m)}}{K_{11}^{(m)} + K_{12}^{(m)}}; \quad \mu_2^{(m)} = \frac{K_{21}^{(m)} - K_{12}^{(m)}}{K_{11}^{(m)} - K_{22}^{(m)}}; \quad \mu_3^{(m)} = \frac{K_{21}^{(m)} - K_{11}^{(m)}}{K_{12}^{(m)} - K_{22}^{(m)}}. \tag{6}$$

All three criteria follow from inequalities $K_{11}^{(m)} + K_{12}^{(m)} \gtreqless K_{21}^{(m)} + K_{22}^{(m)}$, which determine the ratio of the rates of propagation of potentials $\Theta_1^{(m)}$ and $\Theta_2^{(m)}$. This result may be obtained by means of term by term subtraction of the corresponding equations (1) for i, j = 1, 2:

$$\frac{\partial(\Theta_1^{(m)} - \Theta_2^{(m)})}{\partial \mathrm{Fo}} = (K_{11}^{(m)} - K_{21}^{(m)})\nabla^2\Theta_1^{(m)} - (K_{22}^{(m)} - K_{12}^{(m)})\nabla^2\Theta_2^{(m)}. \tag{7}$$

Hence, when $K_{11}^{(m)} + K_{12}^{(m)} = K_{21}^{(m)} + K_{22}^{(m)}$ (which corresponds to $\mu^{(m)} = 1$), Eqs. (1) and (2) are equivalent.

A more accurate analysis permits conclusions to be drawn about coupling between transfer processes when $\mu^{(m)} \gtrless 1$. It may be shown that when $\mu^{(m)} > 1$, the rate of propagation of potential $\Theta_2^{(m)}$ if greater than that for $\Theta_1^{(m)}$, while the contrary holds true when $\mu^{(m)} < 1$.

The choice of one or other of the criteria $\mu_K^{(m)}$ is determined by the signs of the phenomenological coefficients $K_{12}^{(m)}$ and $K_{21}^{(m)}$ in the Onsager equations, but also by the obvious need for a passage to the limit of the form:

$$\mu^{(1)}_{K_{12}=K_{21}=0} = \mathrm{Lu}, \qquad \mu^{(2)}_{K_{12}=K_{21}=0} = \mathrm{Le}. \tag{8}$$

The criterion $\mu_3^{(m)}$ does not satisfy the last condition and must therefore be discarded. It may be shown that when $K_{12}^{(m)} > 0$ and $K_{21}^{(m)} > 0$, the criterion $\mu_2^{(m)}$ must be used, while when $K_{12}^{(m)} < 0$ and $K_{21}^{(m)} < 0$, the criterion $\mu_1^{(m)}$ is physically correct†. Thus, taking account of (2) and (3), we obtain, for heat and mass transfer in a binary gas mixture with (So > 0, Du > 0) and in a porous medium, respectively,

$$\mu_2^{(2)} = \mathrm{Le}(1 + \mathrm{So})/[1 + \mathrm{DuLe}(1 + \mathrm{So})], \tag{9}$$

$$\mu_1^{(1)} = \mathrm{Lu}(1 + \mathrm{Ko}^*)/[1 + \mathrm{LuPn}(1 + \mathrm{Ko}^*)]. \tag{10}$$

Thus, in conformity with (9) and (10), the rates of variation of the heat and mass transfer potential fields are determined not only by the Lu and Le numbers but by the Ph and Ko* numbers for heat and mass transfer in porous materials and by the Du and So numbers for heat and mass transfer in binary gas mixtures. In this case μ^1 = Lu in the absence of thermal gradient mass transfer and phase transitions. Similarly, μ^2 = Le in the absence of thermal diffusion and the Dufour effect. This confirms the earlier conclusion that the Lu and Le numbers characterize the inertia of potential fields in unrelated transfer processes, i. e., when there are no cross terms in the Onsager equations.

It is to be expected that, when boundary conditions of the second and third kinds are specified, the expression for the inertia criteria will contain the corresponding criteria present in the boundary conditions (Posnov numbers and Kirpichev heat and mass transfer numbers for boundary conditions of the second kind, and Biot heat and mass transfer numbers for conditions of the third kind).

NOTATION

Θ_i – generalized transfer potential; ξ and y – coordinates; K_{ij} – phenomenological transfer coefficients in the Onsager equations; T – temperature; p^* – vapor pressure; u – velocity component; ν – viscosity; t – time; Γ – form factor; Lu – Lykov number; Pn – Posnov number; Le – Lewis number; Du – Dufour number; So – Soret number; Pr – Prandtl number; Fo – Fourier number; Ki – Kirpichev number; Ko – Kossovich number; ε – phase transition parameter; Ko* = ε Ko.

REFERENCES

1. A. V. Lykov and Yu. A. Mikhailov, Theory of Heat and Mass Transfer [in Russian], Gosenergoizdat, 1963.
2. Yu. L. Rozenshtok, Proceedings and Abstracts of Papers of the Second All-Union Conference on Heat and Mass Transfer [in Russian], Minsk, 1964.

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†One may reach the same conclusion by requiring that $\mu^{(m)}$ be greater than zero for all positive values of the arguments.